The intended purpose of this document is to provide teachers with a tool to determine student understanding and suggest instructional moves that may help guide a student forward in their learning of a particular concept or standard. This guide is not an exhaustive list of strategies.

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| **Operations and Algebraic Thinking**  **Multiplicative Comparison** | |
| **Represent and solve problems involving multiplication and division.**  **NC.4.OA.1** Interpret a multiplication equation as a comparison. Multiply or divide to solve word problems involving multiplicative comparisons using models and equations with a symbol for the unknown number. Distinguish multiplicative comparison from additive comparison. | |
| **Not Yet** | **Students that are consistently scoring “Not Yet” on multiplicative comparison tasks may need additional support as they are solving tasks. Support students by:**   * Asking questions to support students in thinking about the problem.   + What is happening in the task? Paraphrase or retell the story.   + Can you draw a picture of that relationship?   + Which amounts are larger or smaller in the comparison?   + Can you create a table to show the relationship between the numbers? How is that relationship related to multiplicative comparisons? * Encouraging students to visualize the problem.   + Ask, “What can we draw or build to help us imagine the problem?”   + Provide student with cubes to model the problem. * Modifying the problem by replacing amounts with simpler numbers. |
| **Next Steps:**   * Some students may need intervention on third grade standards, beginning with NC.3.NF.1 where students equipartition a single whole (rectangle or circle) into fair shares. Students may have difficulty understanding the concept of “ \_\_ times as many”. These students may need to equipartition a whole into parts to visually understand this concept. For example, when sharing a brownie among six people, one person's share would be one-sixth, and the original whole consists of the six one-sixths. Have students shade multiple shares of a whole (example four-sixths). They must be able to understand that this area is called 4/6 and can be proven by counting each sixth or describing the area as 4 times as large as a sixth. * Encourage students to represent the situation with manipulatives such as cubes or counters.   *Anna ate 3 donuts.*    *Javon ate 4 times as many donuts as Anna.*     * Students need to be able to move to simple problems involving multiple wholes. For example, *“Anna ate 3 donuts. Javon ate 4 times as many donuts as Anna. How many donuts did Javon eat?”* Assist with making models to represent the situation. Ask, “What multiplication equation represents this model?”  |  |  |  |  |  | | --- | --- | --- | --- | --- | | Anna | 3 |  |  |  | | Javon | 3 | 3 | 3 | 3 | |

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| **Progressing** | **Students that are consistently scoring “Progressing” understand how to equipartition and visually represent the problem. They often represent the problem correctly; however, they may have difficulty answering the question or representing the situation with an equation.** |
| * Support student in recording an equation to represent the situation.   + Discuss the operation related to this problem. Focus on understanding the problem, rather than looking for key words as they can be misleading.   + Ask, “What information are we trying to find?”   + Ask, “How could we write an equation to represent this?”   + Ask, “How could we use multiplication to solve this problem?” * Model to represent a similar problem. Connect the model to multiplication or division. Using a bar model or number line may bring clarity to the multiplicative comparison.   *Al spied 3 times as many cardinals as blue jays at the city park. If Al observed 24 cardinals, how many blue jays did he see?*   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | 24 cardinals | | |  | 24 cardinals | | | | 1/3 | 1/3 | 1/3 | 8 blue jays |  |  |     24 ÷ 3 = 8 or of 24 = 8   * If students are unable to write a correct equation, have them talk about the action of the problem and focus on the repetitive addition or multiplication that is taking place. * Students that are progressing may need assistance understanding a common misconception about equations expressing multiplicative comparisons. The problem is stated, *“The donut shop makes three times as many donuts as pastries.”*  Students of all ages tend to code this directly as 3D = P rather than correctly as 3P = D. The use of a table of values can assist in avoiding having students develop such a misconception by recognizing that the first sentence expresses a relationship not an equation. * Have students match problems and models. Students can determine and discuss which models accurately represent the problem.   Which model accurately models the situation?  *A cheetah runs 3 times as fast as a house cat. If a house cat runs 42 km/h, how fast does a cheetah run?*  42 ÷ 3 = 14 km/h   |  |  |  | | --- | --- | --- | | 42 km/h | | | | 14 | 14 | 14 |   42 x 3 = 126 km/h   |  |  |  |  | | --- | --- | --- | --- | | House Cat | 42 km/h |  | | | Cheetah | 42 | 42 | 42 | |

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| **Meets Expectation** | **Students that are consistently scoring “Meets Expectation” on multiplicative comparison tasks may still need to work on developing fluency, appropriate explanations, and effectively moving between strategies.** |
| **Next Steps:**   * Ask, “How do you know your solution is accurate and reasonable?” * Explain that it’s often helpful to solve a problem a second time using a different strategy to check the accuracy of your solution. Challenge student to solve this problem a second time using a different representation or strategy. A bar model or number line would be a great visual strategy here. Representations may be concrete tools, drawings, or symbols (such as equations). |