The intended purpose of this document is to provide teachers with a tool to determine student understanding and suggest instructional moves that may help guide a student forward in their learning of a particular concept or standard. This guide is not an exhaustive list of strategies.

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| **Operations and Algebraic Thinking****Multiplicative Comparison** |
| **Represent and solve problems involving multiplication and division.****NC.4.OA.1** Interpret a multiplication equation as a comparison. Multiply or divide to solve word problems involving multiplicative comparisons using models and equations with a symbol for the unknown number. Distinguish multiplicative comparison from additive comparison. |
| **Not Yet**  | **Students that are consistently scoring “Not Yet” on multiplicative comparison tasks may need additional support as they are solving tasks. Support students by:*** Asking questions to support students in thinking about the problem.
	+ What is happening in the task? Paraphrase or retell the story.
	+ Can you draw a picture of that relationship?
	+ Which amounts are larger or smaller in the comparison?
	+ Can you create a table to show the relationship between the numbers? How is that relationship related to multiplicative comparisons?
* Encouraging students to visualize the problem.
	+ Ask, “What can we draw or build to help us imagine the problem?”
	+ Provide student with cubes to model the problem.
* Modifying the problem by replacing amounts with simpler numbers.
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| **Next Steps:*** Some students may need intervention on third grade standards, beginning with NC.3.NF.1 where students equipartition a single whole (rectangle or circle) into fair shares. Students may have difficulty understanding the concept of “ \_\_ times as many”. These students may need to equipartition a whole into parts to visually understand this concept. For example, when sharing a brownie among six people, one person's share would be one-sixth, and the original whole consists of the six one-sixths. Have students shade multiple shares of a whole (example four-sixths). They must be able to understand that this area is called 4/6 and can be proven by counting each sixth or describing the area as 4 times as large as a sixth.
* Encourage students to represent the situation with manipulatives such as cubes or counters.

*Anna ate 3 donuts.*  *Javon ate 4 times as many donuts as Anna.** Students need to be able to move to simple problems involving multiple wholes. For example, *“Anna ate 3 donuts. Javon ate 4 times as many donuts as Anna. How many donuts did Javon eat?”* Assist with making models to represent the situation. Ask, “What multiplication equation represents this model?”

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| --- | --- | --- | --- | --- |
| Anna | 3 |  |  |  |
| Javon | 3 | 3 | 3 | 3 |

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| **Progressing** | **Students that are consistently scoring “Progressing” understand how to equipartition and visually represent the problem. They often represent the problem correctly; however, they may have difficulty answering the question or representing the situation with an equation.** |
| * Support student in recording an equation to represent the situation.
	+ Discuss the operation related to this problem. Focus on understanding the problem, rather than looking for key words as they can be misleading.
	+ Ask, “What information are we trying to find?”
	+ Ask, “How could we write an equation to represent this?”
	+ Ask, “How could we use multiplication to solve this problem?”
* Model to represent a similar problem. Connect the model to multiplication or division. Using a bar model or number line may bring clarity to the multiplicative comparison.

 *Al spied 3 times as many cardinals as blue jays at the city park. If Al observed 24 cardinals, how many blue jays did he see?*

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| --- | --- | --- |
| 24 cardinals |  | 24 cardinals |
| 1/3 | 1/3 | 1/3 | 8 blue jays |  |  |

 24 ÷ 3 = 8 or $\frac{1}{3}$ of 24 = 8* If students are unable to write a correct equation, have them talk about the action of the problem and focus on the repetitive addition or multiplication that is taking place.
* Students that are progressing may need assistance understanding a common misconception about equations expressing multiplicative comparisons. The problem is stated, *“The donut shop makes three times as many donuts as pastries.”*  Students of all ages tend to code this directly as 3D = P rather than correctly as 3P = D. The use of a table of values can assist in avoiding having students develop such a misconception by recognizing that the first sentence expresses a relationship not an equation.
* Have students match problems and models. Students can determine and discuss which models accurately represent the problem.

 Which model accurately models the situation?*A cheetah runs 3 times as fast as a house cat. If a house cat runs 42 km/h, how fast does a cheetah run?* 42 ÷ 3 = 14 km/h

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| 42 km/h |
| 14 | 14 | 14 |

 42 x 3 = 126 km/h

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| House Cat | 42 km/h |  |
| Cheetah | 42 | 42 | 42 |

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| **Meets Expectation**  | **Students that are consistently scoring “Meets Expectation” on multiplicative comparison tasks may still need to work on developing fluency, appropriate explanations, and effectively moving between strategies.**  |
| **Next Steps:** * Ask, “How do you know your solution is accurate and reasonable?”
* Explain that it’s often helpful to solve a problem a second time using a different strategy to check the accuracy of your solution. Challenge student to solve this problem a second time using a different representation or strategy. A bar model or number line would be a great visual strategy here. Representations may be concrete tools, drawings, or symbols (such as equations).
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