**Equal Area & Perimeter: Is it Possible?**

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| In this lesson, students consider the relationship between area and perimeter by trying to decide whether a shape can have a perimeter and area that are numerically the same. |

**NC Mathematics Standards:**

**Measurement and Data**

**NC.4.MD.3** Solve problems with area and perimeter.

* Find areas of rectilinear figures with known side lengths
* Solve problems involving a fixed area and varying perimeters and a fixed perimeter with varying areas
* Apply the area and perimeter formulas for rectangles in real world and mathematical problems

**Standards for Mathematical Practice:**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Constructs viable arguments and critiques the reasoning of others.

6. Attend to precision.

7. Look for and make use of structure.

**Student Outcomes:**

* I can apply the area formula for rectangles.
* I can apply the perimeter formula for rectangles.

**Math Language:**

* area
* perimeter
* square units
* length
* width
* dimensions

**Materials:**

* color tiles
* graph paper

**Advance Preparation**:

* Gather color tiles and graph paper for students to use as needed

**Launch:**

1. Introduction (10-15 minutes)

Display the following shapes for students and ask them to find the area and perimeter of each of the rectangles being displayed.

 

Ask students to share the perimeter and area that they came up with. Point out that the first rectangle has an area that is greater than its perimeter, while the second rectangle has an area that is less than its perimeter.

**Explore:**

1. Discovery (20-30 minutes)

Pose the following question to the students. “Is it possible to have a rectangle that has the same numerical area and perimeter?” Set students off to work on this task. Encourage partner work in order to share ideas on how to accomplish the task. Circulate and make note of the reasoning students are using to approach the task.

Look for the following strategies:

* Fix one attribute (area, perimeter, side length) while varying the others using trial and error.
* Fix one attribute (area, perimeter, side length) and use a table to organize possibilities for the other attributes.

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| Area | Perimeter |
| 24 | 6+6+4+4 = 2012+12+2+2 = 288+8+3+3 = 22 |

* Fix one attribute and using an algebraic equation to solve for the other attribute (area = 20, so side 1 + side 2 + side 3 + side 4 = 20).
* Create an algebraic expression for both area and perimeter and insert values into the resulting equations until they are equal (length x width = side 1+ side 2 + side 3 + side 4).

For students who are struggling, choose any of the following questions to give them an entry point:

* What is the same about the two rectangles we looked at earlier?
* What could you change about each rectangle?
* What does area and perimeter change as you change the dimensions of the rectangle?

Push students who have used color tiles to find a rectangle that fits the criteria to then move to using an equation. “How could you represent what you have shown with your color tiles as an equation?”

Correct Solutions: 4 x 4 and 6 x 3

**Discuss:**

1. Debrief & Apply (15-20 minutes)

Once students have adequate time to come up with rectangles that meet the given criteria, come together to discuss. Have students share the dimensions of the rectangles that they have found with the same numerical area and perimeter as well as the different strategies used to find the rectangles (see above for examples of strategies). Discuss the efficiency of the strategies shared. Which ones would help us organize our thinking? Close the lesson by using the formal evaluation/exit ticket suggestion below.

**Evaluation of Student Understanding:**

**Informal Evaluation:**

* Monitor students to see what strategies they are using to solve during the discovery phase of the lesson. Is the strategy efficient?

**Formal Evaluation/Exit Ticket:**

* After the discussion of efficient strategies in the debrief section of the lesson, have students generate 3 rectangles that have the same area, but different perimeters, as well as generate 3 rectangles that have the same perimeter, but different areas.

**Meeting the Needs of the Range of Learners:**

**Interventions:**

* Create a matching game of 24 cards (teacher or student created). The set of cards should contain 8 sets of 3 cards. One card in each set should contain an area measurement, one card a perimeter measurement, and the third card a set of dimensions that are related. Shuffle the cards and have students sort to identify which cards match. Have them discuss how they know each goes together.
* Tell students you are thinking of a rectangle. As them to figure out the dimensions if:
* The area is 36 and the perimeter is 24
* The area is 36 and the perimeter is 26
* The area is 36 and the perimeter is 30
* The area is 36 and the perimeter is 40

Record the solutions on the board and ask students what they notice about the relationship between the area, perimeter, and dimensions. Have them build each rectangle with color tiles and discuss how the changing dimensions affect the look of the rectangle, as well as the area and perimeter.

Repeat the process, but keep the perimeter fixed. Ask students to find the dimensions if:

* The perimeter is 24 and the area is 11
* The perimeter is 24 and the area is 20
* The perimeter is 24 and the area is 27
* The perimeter is 24 and the area is 36

(Students should notice that the length and width added together should total up to half of the perimeter)

**Extension:**

* Have students explore whether or not it is possible to find rectilinear figures that numerically have the same area and perimeter.

\*This lesson was adapted from “Can They Be Equal” <http://nrich.maths.org/6398/note>