**Multiplicative Comparison Models**

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| In this lesson, students will learn about the three types of multiplicative comparison problems and use tape diagrams/bar models to understand how to solve the problems. |

**NC Mathematics Standards:**

 **Operations and Algebraic Thinking**

**NC.4.OA.1** Interpret a multiplication equation as a comparison. Multiply or divide to solve word problems involving multiplicative comparisons using models and equations with a symbol for the unknown number. Distinguish multiplicative comparison from additive comparison.

**Standards for Mathematical Practice:**

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

**Student Outcomes:**

* I can use models to solve multiplicative comparison problems.
* I can use multiplication and division to solve multiplicative comparison problems.
* I can understand the three types of multiplicative comparison problems.

**Math Language:**

* multiplicative comparison
* tape diagram/bar model

**Materials:**

* presentation
* manipulatives (snap cubes, square tiles, graph paper, etc.)
* paper/pencil

**Advance Preparation**:

* Have math manipulatives available for use

**Special Notes:**

* It is recommended to use this lesson for two to three days in order to allow for deeper discussion and understanding.
* Questions and ideas came from <https://www.k-5mathteachingresources.com/support-files/word-problems-multiplicative-comparison.pdf>

**Part 1:**

**Launch:**

1. Introduce Problem (5 minutes)

Display the problem below. Provide students with manipulatives such as square tiles, snap cubes, and/or graph paper and have them use the manipulatives to show what is happening in the problem.

*Mike ran 4 laps around the football field. Tommie ran 5 times as many laps around the football field as Mike. How many laps did Tommie run?*

**Explore:**

1. Solving the Problem (10 – 15 minutes)

Allow students time to work individually and then with partners in order to solve the task. As students work, observe students to see how they are solving the task. Encourage students to share their strategies with one another and describe how they are answering each question.

 Observe:

* + How are students organizing and representing their thinking?
	+ How do students make sense of the question?
	+ How do students use the manipulatives to show their understanding of the problem?
	+ What vocabulary are students using as they solve the task?

Carefully select students to present to the class. Look for students who modeled the problem and used the manipulatives to solve the problem. Be sure to choose students that are able to explain their solutions and how they solved the problem. Choose students that can help extend the understanding of multiplicative comparisons.

**Discuss:**

1. Discussion of Solutions (25 – 30minutes)

Bring the group back together and have selected students share their strategies for solving the task. Show all the manipulatives solutions (snap cubes, graph paper, square tiles, etc.)

 Possible points to address and questions to ask:

* How does your manipulative visually show all the pieces of the problem? (i.e. Where are Mike’s laps? Where are Tommie’s laps?)
* Did Tommie run more or less than Mike?
* Can you draw a picture that would show what your manipulative shows?

If students are unfamiliar with a tape diagram or bar model, show students how the diagram or model could be used to solve the problem.

* Draw one bar that represents Mike and label it with how far he ran. (4 laps)
* If Tommie ran 5 times farther, how many bars would he need to represent him? (5 of Mike’s bars would be used to represent Tommie’s.)
* How could we represent the problem with an equation? What operation do you think we need to use? (4 x 5 = 20, Tommie ran 20 laps.)

 Mike

4 laps

4 laps

4 laps

4 laps

4 laps

4 laps

Tommie

 5 groups of 4 laps = the amount of Tommie’s laps

 5 x 4 = t t = 20 laps

Create an anchor chart. This type of multiplicative comparison is called Large Part Unknown or Unknown Product, because what is missing is the larger number or the final product. You can multiply in order to determine the part that is unknown. Record this on an anchor chart. (Note: Problem on example anchor chart is different.)



When finished, give students several problems to model with manipulatives and a tape diagram/bar model.

* *Kayla has 5 strawberries to use to make a pie. Geneva needs 2 times as many strawberries to make her pie. How many strawberries does Geneva need to make her pie?*
* *Giovanni is 2 years old. His brother Paul is 4 times older than him. How old is his brother Paul?*
* *Johnny has 9 toy cars. Hayden has 4 times as many toy cars as Johnny. How many toy cars does Hayden have?*

**Part 2:**

**Launch:**

1. Introduce Problem (5 minutes)

Display the problem below. Provide students with manipulatives such as square tiles, snap cubes, and/or graph paper and have them use the manipulatives to show what is happening in the problem.

*An extra-large pizza is $24 and costs 3 times as much as a small pizza. How much does a small pizza cost?*

**Explore:**

2. Solving the Problem (10 – 15 minutes)

Allow students time to work individually and then with partners in order to solve the task. As students work, observe students to see how they are solving the task. Encourage students to share their strategies with one another and describe how they are answering each question.

 Observe:

* + How are students organizing and representing their thinking?
	+ How do students make sense of the question?
	+ How do students use manipulatives to show their understanding of the problem?
	+ What vocabulary are students using as they solve the task?

Carefully select students to present to the class. Look for students who modeled the problem with drawings or manipulatives. Be sure to choose students that are able to explain their solutions and how they solved the problem. Choose students that can help extend the understanding of multiplicative comparisons.

**Discuss:**

1. Discussion of Solutions (25 – 30 minutes)

Bring the group back together and have selected students share their strategies for solving the task. Show various models and solutions (snap cubes, graph paper, square tiles, etc.)

 Possible points to address and questions to ask:

* How does your model visually show all the pieces of the problem? (i.e. Where is the cost of the extra large pizza? Where is the cost of the small pizza?)
* Does the extra large pizza cost more or less than the small pizza?
* Where in your diagram do you see the cost of the extra large pizza? The small pizza?
* Can you draw a picture that would show what your manipulative shows?

Additional questions to help students make sense of the drawing or model:

* Which item costs more? (extra-large pizza)
* How much more does it cost? (3 times as much)
* What don't we know? (how much the small pizza cost) How do we find it out?
* How could we represent the problem with an equation? What operation do you think we need to use? ($24 ÷ 3 = ? or 3 x ? = $24 A small pizza costs $8.)

$24

Extra Large Pizza

?

 Small Pizza

Create an anchor chart. This type of multiplicative comparison is called Smaller Part Unknown or Group Size Unknown, because what is missing is the smaller number or the group size. Students may use a division or multiplication equation in order to solve the problem. Record this on an anchor chart. (Note: Problem on example anchor chart is different.)



When finished, give students several problems to model with manipulatives and a tape diagram/bar model.

* *An apple pie cost $16. A pecan pie cost 4 times less than the apple pie. How much does the pecan pie cost?*
* *There are 15 gorillas at the zoo. There are three times as many gorillas as wolves. How many wolves are there at the zoo?*
* *Tyler and Jackson went fishing. Jackson caught 36 fish. He caught four times as many fish as Tyler. How many fish did Tyler catch?*

**Part 3:**

**Launch:**

1. Introduce Problem (5 minutes)

Display the problem below. Provide students with manipulatives such as square tiles, snap cubes, and/or graph paper and have them use the manipulatives to show what is happening in the problem.

*A single balloon costs $3 and a bunch of balloons costs $12. How many times as much does the bunch of balloons cost than the single balloon?*

**Explore:**

1. Solving the Problem (10 – 15 minutes)

Allow students time to work individually and then with partners in order to solve the task. As students work, observe students to see how they are solving the task. Encourage students to share their strategies with one another and describe how they are answering each question.

 Observe:

* + How are students organizing and representing their thinking?
	+ How do students make sense of the question?
	+ How do students use the manipulatives to show their understanding of the problem?
	+ What vocabulary are students using as they solve the task?

Carefully select students to present to the class. Look for students who modeled the problem using models such as drawings or manipulatives. Be sure to choose students that are able to explain their solutions and how they solved the problem. Choose students that can help extend the understanding of multiplicative comparisons.

**Discuss:**

1. Discussion of Solutions (15 – 25 minutes)

Bring the group back together and have selected students share their strategies for solving the task. Show various models and solutions (snap cubes, graph paper, square tiles, etc.)

 Possible points to address and questions to ask:

* How does your model visually show all the pieces of the problem? (i.e. Where is the single balloon? Where is the bunch of balloons?)
* Does the single balloon cost more or less than the bunch of balloons?
* Where in your diagram do you see the cost of the single balloon? The bunch of balloons?
* How many times more is the bunch of balloons than the single balloon?
* Can you draw a picture that would show what your manipulative shows?

Additional questions to help students make sense of the drawing or model:

* How much does each bar represent? ($3)
* If the red bar represents the bunch and it has to cost $12, how many bars will you need? (3) How many times more does it cost? (3 times more)
* How could we represent the problem with an equation? What operation do you think we need to use? ($12 ÷ $3 = ? or ? x $3 = $12)

$3

 Single Balloon

$12

Bunch of Balloons

Create an anchor chart. This type of multiplicative comparison is called Multiplier Unknown or Number of Groups Unknown, because what is missing is how many times you have to multiply or the number of groups. Students may use division or multiplication in order to determine the missing amount or what you multiply by. Record this on an anchor chart. (Note: Problem on example anchor chart is different.)



When finished, give students several problems to model with manipulatives and a tape diagram/bar model.

* *A lizard weighs 4 pounds. A rattlesnake weighs 20 pounds. How many times more does lizard weigh than the rattlesnake?*
* *A single pencil cost nine cents. A pack of pencils costs ninty cents. How many times more is the pack of pencils than the single pencil?*
* *For lunch, Cassandra ate 7 grapes. For dinner, Cassandra ate 35 grapes. How many times more grapes did Cassandra eat a dinner than lunch?*

Close the lesson by displaying the three opening problems. Have students discuss how the problems are alike and how they are different. Review the three parts of the problem (product, group size, and number of groups) and the three types of multiplicative comparison problems.

* *Mike ran 4 laps around the football field. Tommie ran 5 times as many laps around the football field as Mike. How many laps did Tommie run?*
* *An extra-large pizza is $24 and costs 3 times as much as a small pizza. How much does a small pizza cost?*
* *A single balloon costs $3 and a bunch of balloons costs $12. How many times as much does the bunch of balloons cost than the single balloon?*

**Evaluation of Student Understanding:**

**Informal Evaluation:**

* Can students make a visual model to show what the question is asking?
* Can students make the connection between multiplication and division?
* Can students understand the different types of multiplicative comparison problems?

**Formal Evaluation/Exit Ticket:**

* Exit tickets provided at the end of each day.

**Meeting the Needs of the Range of Learners:**

**Interventions:**

* Encourage students to use manipulatives and visual models to help them understand the problems.

**Extensions:**

* Have students write their own multiplicative comparison problems for each type of problem.

**Possible Misconceptions/Suggestions:**

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| **Possible Misconceptions** | **Suggestions** |
| * Students do not understand the problem.
* Students are struggling to draw a diagram that matches the problem.
 | * Give students manipulatives and have them create a display that would show what is happening in the problem.
* Have students determine the different parts of the problem – what is known and what is unknown.
* Continue practicing using manipulatives and models. Have students show how each part of the problem matches the diagram. (Graph paper might help as well.)
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**Possible Solutions:**

**Day 1 Practice Problems:**

* Kayla has 5 strawberries to use to make a pie. Geneva needs 2 times as many strawberries to make her pie. How many strawberries does Geneva need to make her pie? (5 x 2 = 10 strawberries)
* Giovanni is 2 years old. His brother Paul is 4 times older than him. How old is his brother Paul? (2 x 4 = 8 years old)
* Johnny has 9 toy cars. Hayden has 4 times as many toy cars as Johnny. How many toy cars does Hayden have? (9 x 4 = 36 toy cars)

**Day 2 Practice Problems:**

* An apple pie cost $16. A pecan pie cost 4 times less than the apple pie. How much does the pecan pie cost? ($16 ÷ 4 = $4 or 4 x ? = $16)
* There are 15 gorillas at the zoo. There are three times as many gorillas as wolves. How many wolves are there at the zoo? (15 ÷ 3 = 5 wolves or 3 x ? = 15)
* Tyler and Jackson went fishing. Jackson caught 36 fish. He caught four times as many fish as Tyler. How many fish did Tyler catch? (36 ÷ 4 = 9 fish or 4 x ? = 36)

**Day 3 Practice Problems:**

* A lizard weighs 4 pounds. A rattlesnake weighs 20 pounds. How many times more does lizard weigh than the rattlesnake? (20 ÷ 4 = 5 times more or ? x 4 = 20)
* A single pencil cost nine cents. A pack of pencils costs ninty cents. How many times more is the pack of pencils than the single pencil? (90 ÷ 9 = 10 times more or ? x 9 = 90)
* For lunch, Cassandra ate 7 grapes. For dinner, Cassandra ate 35 grapes. How many times more grapes did Cassandra eat a dinner than lunch? (35 ÷ 7 = 5 times more or ? x 7 = 35)