The intended purpose of this document is to provide teachers with a tool to determine student understanding and suggest instructional moves that may help guide a student forward in their learning of a particular concept or standard. This guide is not an exhaustive list of strategies.

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| **Number and Operations in Base Ten** **Addition and Subtraction** |
| **NC.4.NBT.4** Add and subtract multi-digit whole numbers up to and including 100,000 using the standard algorithm with place value understanding.  |
| **Not Yet**  | **Students that are consistently scoring “Not Yet” on addition and subtraction tasks could have a variety of errors. *Fourth Grade is the first grade level in which students are expected to be proficient at using the standard algorithms for addition and subtraction. However, other previously learned strategies are still appropriate for students to use* - as long as students have a strategy that is efficient and effective in getting a sum or difference. Students consistently scoring “Not Yet” do not have any effective strategy for addition or subtraction or are struggling with the difference between addition and subtraction. The more comfortable students become with estimating the sum or the difference, the better they will be able to decide if their answer is reasonable.** |
| **Next Steps:****For students having trouble identifying the difference between addition and subtraction:*** Review terminology and meanings for both addition (join, sum, add) and subtraction (separate/take away, difference, subtract, comparisons).
	+ Students may have a strong reliance on “key words” (i.e. more, less, total, altogether, left) which is a common misconception and is misleading to students as they solve word problems.
	+ Key words are counterintuitive to their problem solving ability.
* Have students to make models or pictorial representations of the problem.
* After students can successfully make models, support students in recording an equation to represent what is occurring in the problem or their picture.
* Support students to create their own real world situations in which addition and /or subtraction would need to be used.

 **For students having difficulty using an effective strategy to add or subtract:*** Move students back to two digit numbers and have students solve a two digit addition or subtraction problem using their strategy.
* Start with models (using base ten blocks, Cuisenaire rods, counters), move to pictorials representations, and then move to symbols.
* Go back to the basics of place value. Start by asking students:
	+ What is the value of each digit? (i.e. in 52 – students should be able to say the 5 has a value of 5 tens or 50 and the 2 has a value of 2 ones or 2)
	+ How can you decompose this number? How can you decompose this number another way?
	+ Students should be able to break down the number into different varieties. (i.e. 52 – five tens and 2 ones, 4 tens and 12 ones, 3 tens and 22 ones, 2 tens and thirty two ones, one ten and 42 ones, or 52 ones)
	+ Build students up to grouping by tens. Students can use straws or another manipulative that can be bundled together first, then move students into pre-grouped models (i.e. base ten blocks). Being able to model groups of 10 and groups of 100 helps students understand 10 ones as “10” as well as 10 tens as a “hundred.” As students become comfortable bundling tens, move students into bundling hundreds.
	+ After students are comfortable making their groups of 10 and groups of 100, it is important to connect this back to using these groups to solve different addition and subtraction problems using these same manipulatives. Students should be able to begin to solve addition and subtraction problems using their manipulatives. When students have mastered using their manipulatives, they can begin to connect their manipulative models to equations in order to visualize the connections to place value.
	+ Search online for virtual manipulatives such as base ten blocks for addition and subtraction. These are a virtual way for students to connect the manipulatives to the concept of addition/subtraction.

**For students with errors related to counting:*** Provide concrete objects and ask students to model the situation with the objects.
* Students who are using one-to one correspondence in counting may find it helpful to move objects to accurately determine the count.
* As students move from models and pictures, help them to see the difference in numeric representations. The important understanding is that arrangements of the digits in the numeric representations matters as opposed to in concrete models. (i.e. When using base ten blocks you do not have to have the blocks in a particular order for it to represent its value. Yet when using only numbers you cannot change the order of the digits!)
* Use a hundreds chart to help students look for patterns with both addition and subtraction. For example, ask students:
	+ Put your finger on the number 57. What number comes just before 57? What number comes right after?
	+ What number is 10 more than 57? What number is 10 less than 57?
	+ What are some numbers that are greater than 57? What are some numbers less than 57? Are these numbers a lot less/greater than 57 or a little bit less/greater than 57?
	+ Determine simple sums and differences by moving around the chart (i.e. 45+3=;57-4=; 17+5=;51-5=)
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| **Progressing** | **Students that are consistently scoring “Progressing” have a strategy to use for addition and subtraction; however, they have not mastered this strategy or may also still struggle with the difference between addition and subtraction. Students also may not understand how to decide if their answer is reasonable and may need additional work with estimation strategies. The more comfortable students become with estimating the sum or the difference, the better they will be able to decide if their answer is reasonable.** |
| **Next Steps:****For students having trouble identifying the difference between addition and subtraction:*** Review meanings and terminology for addition (sum) and subtraction (difference).
* Have students to make models or pictorial representations of the problem.
* After students can successfully make models, support students in recording an equation to represent what is occurring in the problem or their picture.
* Support students to create their own real world situations in which addition and /or subtraction would need to be used.

**For students with errors related to counting:*** Students may need additional practice with their basic addition and subtraction facts.
* Provide concrete objects and ask them to model the situation with the objects.
* Students who are using one-to one correspondence in counting may find it helpful to move objects to accurately determine the count.

**For students struggling to keep their numbers organized:*** Model for students how to use a place value chart, grid paper, or notebook paper turned sideways. All of these provide grid lines that ensure each number is in its proper place.
* Have students label each place value above the column of numbers. This will help students connect place value to the algorithm so that the numbers are lined up by value.

**For students adding a column when it does not exist/placing more than one digit in a column:** * This shows that students need to work on their understanding of the place value columns and their ability to use regrouping.
* They need to develop their knowledge of decomposing and regrouping. (i.e. That 12 ones is equal to 1 ten and 2 ones.)
* Students also need to understand that only one digit can be in the ones place and that their column represents the ones place.
* Labeling the place values above each column may help students to connect the value of each place to the algorithm.
* Have students model their algorithm using base ten blocks in order to connect regrouping to the numeric form when using the algorithm.

**For students reversing the digits when they regroup, forgetting to add what they have regrouped, or not carrying what they have regrouped into the next column:** * Have students go back to the value of that digit. (i.e. If students have 7 ones and 4 ones when combined, this equals 11. This can be regrouped to 1 ten and 1 one.)
* Model the regrouping process using base ten blocks and a place value grid

**For students subtracting the smaller from the larger number in each column without regards to its place:** * This error can occur when students hear “you must always start with the bigger number and subtract the smaller number” or “you cannot subtract a bigger number from a smaller number”. These statements are inaccurate and can cause misconceptions in later grades.
* Students need to understand that it is possible to subtract a larger number from a smaller number so that they don’t carry a misconception to higher grades. This would result in getting negative numbers.
* Have students go back and focus on the value of each digit.
	+ Ask: “What is the value of this digit?” (Point to a particular digit to determine whether students include the place value. Ex: Point to the “4” in 742 and listen for “4 tens” or “40” rather than “4”.)
* Students need to understand each column is not its own separate problem, but only organizing your number by its place value.
* Strong estimation strategies can help students to catch these errors.
	+ Ask: “How can you check the reasonableness of your results?”
* Go back to base ten blocks or other hands-on manipulatives and have students model the same subtraction problem

**For students struggling with regrouping in subtraction:** * Strong estimation strategies can help students to catch these errors.
	+ Ask: “How can you check the reasonableness of your results?”
* Go back to base ten blocks or hands-on models so that students can physically see what they have to do to when regrouping.
* After modeling using base ten blocks, have students connect their physical models to numeric form.
* Ask students:
	+ How can you take away 18 from 46? How do you know this?
	+ If you have 6 ones, how can you take away 8 ones? Where could you get more ones without changing the 46?
	+ If you’ve regrouped the 46 to 3 tens and 16 ones, do you still have 46? How could you prove this?
	+ Now how would you subtract 18 from your 3 tens and 16 ones?

**For students struggling with subtraction across zeroes:*** Strong estimation strategies can help students catch these errors.
	+ Ask: “How can you check the reasonableness of your results?”
	+ The more comfortable students become with estimating the sum or the difference, the better they will be able to decide if their answer is reasonable.
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| **Meets Expectation**  | **Students that are consistently scoring “Meets Expectation” on addition and subtraction tasks should have a good command of place value and be considered fluent with addition and subtraction. Fluency means students are accurate, flexible, and efficient with their strategies. According to NCTM President Linda Gojak, “Computational fluency refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate flexibility in the computational methods they choose, understand and can explain these methods, and produce accurate answers efficiently. Focusing on efficiency rather than speed means valuing students’ ability to use strategic thinking to carry out computation without being hindered by many unnecessary or confusing steps in the solution process. Accuracy involves considering the meaning of an operation, recording work carefully, and asking oneself whether the solution is reasonable.” Students may need to work on developing mental fluency, appropriate explanations, and effectively moving between strategies in order to be considered fluent.**  |
| **Next Steps:** * Ask students to explain their addition or subtraction process. Students should be able to explain why they regrouped. Their explanation should be founded upon their understanding of place value.
* Ask: “How do you know your solution is accurate and reasonable?” Students should be able to answer this by applying estimation strategies.
* Explain that addition and subtraction are inverse operations. Challenge students to be able to check their work by using an inverse operation.
* Students should be able to solve their problem using a variety of strategies and understand how place value connects their various strategies (i.e. manipulatives, number lines, algorithm).
* Students should be able to select various strategies based on the numbers in the problem. For example, when adding 199 + 199, a student may use doubles, adding 200 + 200 to get 400 and then subtracting the two extra ones in order to get 398. The standard addition algorithm is not needed to solve this problem. Students need to be flexible and strategic when selecting the methods they will use to solve addition and subtraction problems.
* Encourage and teach students to use mental math strategies to solve addition and subtraction problems.
* Use number talks daily to help students become fluent with mental math strategies for adding and subtracting. Discuss why students selected particular strategies and how the numbers influenced their selection process.
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