**Introduction to Writing a Mathematical Argument in Grade 3**

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| In this lesson, students begin to explore ways to construct and write a mathematical argument in the context of addition and subtraction. This is one lesson that addresses the introduction to constructing a mathematical argument, one purpose for mathematical writing. There should be multiple lessons throughout the year to support students with the construction and also the critique of a mathematical argument. |

**NC Mathematics Standards:**

**Number and Operations in Base Ten: Use place value to add and subtract.**

**NC.3.NBT.2** Add and subtract whole numbers up to and including 1,000:

* Use estimation strategies to assess reasonableness of answers.
* Model and explain how the relationship between addition and subtraction can be applied to solve addition and subtraction problems.
* Use expanded form to decompose numbers and then find sums and differences.

**Standards for Mathematical Practice:**

3. Construct viable arguments and critique the reasoning of others.

6. Attend to precision.

**Student Outcomes:**

* I can construct a mathematical argument.
* I can communicate my mathematical thinking clearly.
* I can add and subtract whole numbers up to and including 1,000.

**Math Language:**

* Addition, subtraction, addend, sum, equal, claim, reasons

**Materials:**

* Blank paper, pencil
* manipulatives for students to explore with such as graph paper, sticky notes, unifix cubes, and/or base ten blocks

**Advance Preparation**:

* Have materials available during the explore phase of the lesson.

**Launch:**

1. Introduce students to the idea of a mathematical argument (10-15 minutes)

(The teacher script and student responses are examples of what might be said in class, it does not have to be followed word by word and is only there to give you an idea of how the launch may look in a third grade classroom.)

* Teacher: *Have you ever been in an argument?*
* Teacher: *Typically, when we argue, it’s because we are unhappy. Today, we are going to learn how to write a mathematical argument*. *The purpose of a mathematical argument is to demonstrate that a claim is true or false.*
* Teacher: *The word claim might be new for you, it means a statement or response. I will share one with you and then we will share reasons to prove the claim is true or false.*
* Teacher: *Here is a claim, (select two students in the class) Student A is taller than Student B*.
* Ask: *Is this claim true or false?*
* Students: True
* *Write the claim on chart paper so the class can have a visual representation of an argument (It is true that Student A is taller than Student B).*
* Ask: *How can you prove that its true? (Write the reasons on the chart paper to provide students with a model of the argument.)*
* Students: Because Student A is taller.
* Teacher: *But, how do we know that?*
* Student: Oh, we could measure them!
* Measure each student and write measurements on the board (\**This is a key step in a mathematical argument because the mathematical work supports the claim*).
* Teacher: *How does this information tell us who is taller?*
* Student: Because the person whose height is a greater number of inches means they are taller. (\**This is another key part of the argument because it explains why a measurement of inches can be used to prove who is taller. When students write their own arguments, they may need to be prompted to explain why their mathematical work (i.e., a mathematical procedure, representation) proves the claim to be true or false.*)
* Teacher: *You have stated a claim and provided reasons to prove your claim is true*

You may want to re-state the claim and reasons to help students see the parts of their argument.

* Teacher: *A mathematical argument is a sequence of statements and reasons to demonstrate that a claim is true or false.*
* Share an example from your own class or use the following example to help students understand the purpose and structure of a mathematical argument. This will also provide students with an example as to how they have previously engaged in a mathematical argument.
* Teacher: *Remember when we’ve had problems like 35 – x = 7 and you’ve had to convince me that x = 28. Well, x = 28 was your claim. Remember, a claim is a statement or response. What were some of your mathematical reasons to prove your claim was true?* If no one answers, you can use this example: *Last year, a student said that if you use a number line and start at 35, if you count down to 7 the difference between the numbers is 28 because 35 – 28 = 7. Does anyone else have any other reasons?*
* Students may say, “*I started at 7 and counted up 10 and then 10 more to get to 27 and then I counted up 8 more to get to 35. So, 10 + 10 + 8 = 28 and 35 – 28 = 7.”*
* Teacher: *I am hearing a lot of reasons for why the claim x = 28 is true – it looks like we have already been working on mathematical arguments! Today we are going to write our mathematical argument.*

Introduce the task.

* Tell students that you have found the following mathematical statement from one of your former students, it says 16 + 29 = 43.
* Ask students to work with a partner to convince you if this statement is true or false.

**\*Note:** The idea of “convincing” is an important component to include to set students up to write a mathematical argument. Other questions to include to set students up to construct an argument are “Explain why”; “Do you agree/disagree”; “How do you know \_\_\_ is true?” These statements help position students to include mathematical reasons to support their response to whether the mathematics problem’s answer is true or false.

**Explore:**

1. Writing Mathematical Claims (20 minutes)

Pair students up, provide them with a pencil and blank paper, and also tell them that they can use any of the manipulatives you have laid out to help them figure how to convince your former third grade student if 16 +29 =43 is true or false. Use the questions in table 1 if students are having difficulty engaging with the problem.

Table 1.

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| **Observation** | **Questions to Ask** |
| Students have difficulty engaging with the problem. | * “What do you notice about this statement?” * “What could be a first step to solve?” * “Do you agree with this statement? Why or why not?” |
| Students have difficulty coming up with convincing reasons. | * “How can you prove that your claim is true?” * “What work can you show?” * “What would convince someone your claim is true?” * “Why does your work prove your claim to be true?” |

**Discuss:**

1. Discussing Our Claims (15 minutes)

Bring students together and rewrite the focus question: Is 16 + 29 = 43 true? Ask students to share their ideas about whether this statement is true or false and to provide convincing reasons to support their claim.

Write all student ideas on the board or on chart paper. It is important to recognize and discuss misconceptions about the mathematics content from the class discussion. As student share ideas, organize them on the board with the students’ claim and then their reasons to prove their claim is true or false (Table 2). You may need to encourage students to explain why their claim is true and ask them why their mathematical reasons prove the claim is true.

**\*Note:** *the reasons given in an argument should demonstrate that a claim is true or false. An argument is not an explanation of the steps to solve or rewriting the problem-solving process in words, or providing reasons that are not mathematical (e.g., because my brain told me).*

Table 3 provides some sample questions and possible sentence frames to support the class discussion.

Table 2. Example of a Student Argument

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| **Claim** | 16 + 29 does not equal 43. |
| **Statement of Reasons**  (There are multiple correct reasons for this argument. Students should be able to provide a counterexample for why the statement is not true.) | ***Examples:***  Student A  Because 10 + 20 = 30 and 6 + 9 = 15 and 15 + 30 = 45. 45 is not equal to 43. So, 16 + 29 is not equal to 43.  1  Student B  16  + 29  45  I added 16 +29 and the sum is 45 not 43 so 16+29 is not equal to 43. |

Table 3. Sample Questions to Facilitate Classroom Discourse

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| **Sample Questions** | **Possible Responses or Talk Frames** |
| What is your claim? | * “My claim is \_\_\_\_\_\_\_\_\_\_\_\_.” * “\_\_\_\_\_\_ is true/false.” |
| * What are your reasons for your claim? * (If only a claim is given) Can anyone add-on to prove this claim is true or false? * Explain why. * How do you know why? * Can you convince me that your claim is true/false? * Do you agree or disagree with this idea? | * “\_\_\_\_\_\_\_ is true/false because\_\_\_\_\_.” * “I agree/disagree because \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.” * “I think because \_\_\_\_\_\_\_\_\_\_.” * “I would like to add-on that\_\_\_\_\_\_\_\_\_\_\_” * “The claim is true because \_\_\_\_\_\_\_\_\_\_” |

Leave all student ideas on the board or chart paper and have students return to their tables and write their mathematical argument.

***\**Note:** It is important to leave the student ideas on the board because this is an initial written mathematical argument. The ideas will help provide a scaffold for students when composing their individual argument. However; it is also important to remind students not to directly copy an argument from the board.

**Evaluation of Student Understanding:**

**Informal Evaluation:**

* Observe students and ask questions as they are identifying mathematical reasons to prove the statement as true or false.
* Observe students during the mathematical discussion to identify who may be understanding the purpose of a mathematical argument (to demonstrate the truth or falsity of a claim).

**Formal Evaluation:**

* Students’ mathematical writing at the conclusion of the discussion.
* Aspects to attend to consider when evaluating and providing feedback to students:
  1. Is there a claim clearly stated?
  2. Are the individual reasons to prove the claim true/false?
  3. Does the sequence of reasons make logical sense?
  4. Is the argument written with attention to precision?

\***Note:** Question 4 addresses the Standard for Mathematical Practice 6: Attend to precision. Because the audience for a mathematical argument is someone other than author, precise mathematical language is an important consideration. However; this should not prevent students from engaging in mathematical argumentation. Students may write their argument and then revise it.

Optional Scoring Rubric for 16 + 29 = 43

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| **Category** | 0 | 1 | 2 |
| Claim | No claim is present or the claim is incorrect.  “16+29 = 43 is true.” | The claim is part of the response but is unclear.  “I added 16 + 29 and got an answer of 45 so the sum is not 43.” | The claim is stated.  “16 + 29 does not equal 43.” |
| Statement of Reasons | No reasons are stated or the reasons do not prove the claim to be true.  “I added.”  “20 – 10 = 10 and 9 – 6 = 3 so 13 is the answer.” | Some reasons are missing and/or there are mathematical errors.  “10 + 20 = 30 and 6 + 9 = 15 and 30 + 15 = 45.” | The reasons prove the claim to be true or false.  “29 + 10 = 39; 39 + 6 = 45. So, 16 +29 = 45 not 43.” |
| Attention to Precision | The writing is unclear. | The writing is somewhat clear. | The ideas in the argument are clearly stated. |

**Meeting the Needs of the Range of Learners:**

**Interventions:**

* For students who may have difficulty writing:
  + Include sentence frames such as:
    - “The statement is \_\_\_\_\_\_\_ because \_\_\_\_\_\_\_\_.”
    - “I agree with \_\_\_\_\_\_ because \_\_\_\_\_.”
    - “My claim is \_\_\_\_\_\_\_\_\_. My reasons are \_\_\_\_\_\_\_\_.”
  + Students can share their argument aloud while another person writes their argument.

\*Note: it is important that all students have an opportunity to engage in mathematical argumentation as this is an important process for thinking about mathematical ideas.

**Extensions:**

* Students can critique a partner’s argument. (This could also be a follow-up lesson for this activity).
* Students can pose a related problem.

**Special Notes:**

* This offers one way of introducing written mathematical argumentation. The same type of lesson could be used with other mathematics content. A key component is that students need to be positioned to craft a mathematical argument.
* Students should have familiarity with the mathematical content to write a mathematical argument. This will help them identify reasons to support their claim.
* Developing a sequence of reasons for the argument is often the most difficult for students. If students are struggling to identify the reasons it may be indicative of a lack in understanding about the mathematical concept.